Prestack amplitudes forward modeling using lithofacies information

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SUMMARY

This paper proposes a forward modeling of pre-stack amplitudes using lithofacies defined by probability vectors with the help of compositional data concepts and Shepard interpolation. The algorithm is demonstrated on real seismic data with good results.

INTRODUCTION

Generally, offset or angle-limited seismic traces are modeled using rock physics equations. The forward modeling computes the reflectivity volume using the anisotropic Zoeppritz equations (Sheriff) or some linearization of this (such as Aki-Richards or Shuey approximation). This reflectivity cube is then convolved with appropriate wavelet, generating 3D cubes of seismic amplitude response. We propose here an alternative approach using Democratic Neural Network Association results (Hami-Eddine, 2009) that are couples of probability vectors and amplitude collections. The algorithm is demonstrated on real seismic data.

METHODOLOGY

Into the Democratic Neural Network Association (DNNA) method, a lithofacies is represented by several couples [probability vector, amplitude vector] where probability vectors approach as closely as possible boundaries of each lithofacies,i.e. transitions between lithofacies..

Basic mathematical operations on probability vectors are not trivial because probability space does not have a natural Euclidean frame. More precisely, for the purpose of the work, we should be able to evaluate the distance between two probability vectors for doing amplitude interpolation. Because Euclidean geometry axioms cannot be used within a probability space without obtaining contradictions, measuring distances between two probability vectors is an issue. Other problems might appear in many situations, like those where results end up outside the probability space, e.g. when translating probability vectors. For those reasons, Euclidean geometry is not suitable for analyzing probability vectors. Considering probability vectors as compositional data and using the Aitchinson transform to map probability vectors into a Euclidean multidimensional space enable us to perform amplitude interpolation using the Shepard technique. Moreover, we plan to use this work for lithofacies inversion from prestack amplitude and performing it using probability space would imply to take into account additional optimization constraints.

Compositional data

Compositional data is implicitly stated in the units, as they are parts of a whole, like volume percent, ppm or molar properties. The most common examples have a constant sum κ and are also called closed data. Probability vector is a particular case where $\kappa = 1$. They are considered as real vectors having positive components and these vectors span the standard simplex, defined as

$$S^d = \left\{ \mathbf{p} = [p_1, \dots, p_d] \,|p_i > 0, i = 1, \dots, d;\ \sum_{i=1}^{d} p_i = 1 \right\}$$

Figure 1 below shows one element $\mathbf{p}$ of S^3 with $\mathbf{p} = [p_1, p_2, p_3] = [0.5, 0.20, 0.30]$.This particular representation when *d*=3 is named ternary diagram.

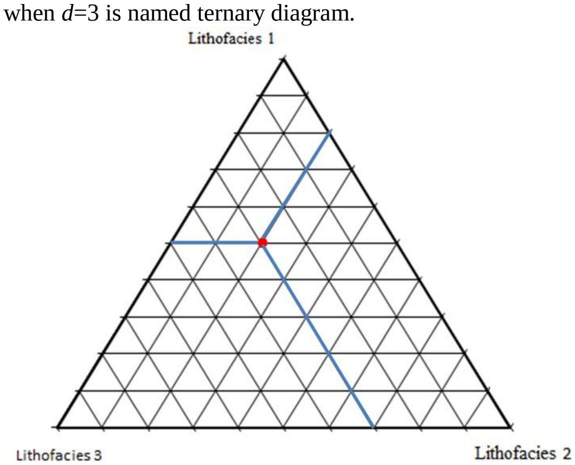

Figure 1: The lithofacies probability vector **p** displayed within a ternary diagram: the *i*-th corner is associated to the *i*-th lithofacies. Each corner of the triangle corresponds to a probability one; each edge corresponds to a null probability of the opposite edge. The intersection of the dotted lines gives the location of **p.**

Aitchison transform

The Aitchinson transform (Aitchinson,1982) is based upon transforming the data using log-ratios, an approach called log-ratio analysis (LRA). The motivation behind LRA is that compositional data carry only relative information about the components and hence working with logs of ratios is appropriate for analyzing this information. A popular log-ratio transform, suggested by (Aitchison, 1983), is the centered log-ratio transform defined by

$$\mathbf{q} = [q_1, \dots, q_d] = [\log(p_i)\,/g(\mathbf{p})]_{i=1,\dots,d}$$

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where $g(\mathbf{p})$ stands for the geometric mean of components of vector $\mathbf{p}$. This transform maps the standard simplex S^d onto a d-dimensional subspace of $\mathbb{R}^d$, given by

$$Q^d = \left\{ \mathbf{q} = [q_1, \dots, q_d] \mid ; \sum_{i=1}^{d} q_i = 0 \right\}$$

The so called isometric log-ratio (Egozcue et al., 2003) is then given by $\mathbf{m} = \mathbf{Hq}$, where $\mathbf{H}$ is a $(d$-1)× d orthonormal matrix whose rows are orthogonal to $\mathbf{1}_d$, the d-vector of ones. A standard choice of $\mathbf{H}$ is the Helmert sub-matrix obtained by removing the first row from the Helmert matrix with general coefficient

$$\mathbf{H}_{ij} = \begin{cases} \dfrac{1}{\sqrt{d(d-i+1)}} \text{ if } i \le j \\ -\dfrac{1}{\sqrt{d(d-i+1)}} \text{ if } i = j+1 \\ 0 \text{ otherwise} \end{cases}$$

These formulas need to be adapted when probability vector has zero components. We consider instead the slightly modified vector with components:

$$p_i^* = \begin{cases} \varepsilon \;\text{ if } p_i = 0 \\ p_i - \varepsilon(d-k)/k \;\text{ otherwise} \end{cases}$$

where k is the number of non-zero components and ε a small positive number.

Prestack forward modeling using lithofacies

Prestack amplitudes forward modeling consists in determining from lithofacies information, a collection of amplitudes corresponding from various distance between source and receiver (offset) reflecting on the same interface.

These collections are considered as amplitude vectors $\mathbf{A}^j, j = 1, \dots, n$. Lithofacies considered as a categorical variable (without natural ordering) of geological deposits is replaced here by a probability vector $\mathbf{p}^j, j = 1, \dots, n$, with the highest value corresponding to the most probable facies index. The correspondence between lithofacies and amplitude collections is here taken as a result of a supervised classification obtained with the DNNA giving n neurons that are specific couples (each couple being a representative of a lithofacies) of probability vector-amplitude vector $\left(\mathbf{p}^j, \mathbf{A}^j\right)_{j=1,\dots,n}$ where $\mathbf{p}^j \in S^d$ and $\mathbf{A}^j \in \mathbb{R}^p$. (Figure 2).

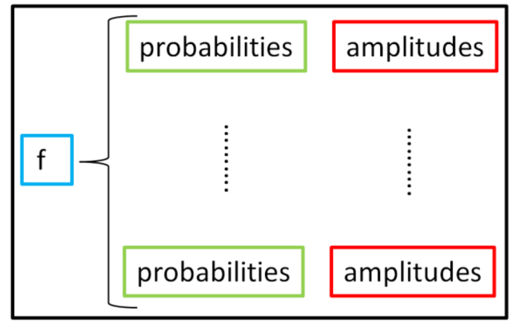

Figure 2: Correspondence lithofacies-prestack amplitude collection.

The proposed forward modeling is based on following steps:

i. starting from mapping each probability vector $\mathbf{p}^j$ into Euclidian space, giving a vector $\mathbf{m}^j \in \mathbb{R}^{d-1}$,
ii. performing amplitude interpolations using the Shepard interpolation with Euclidian distance.

Shepard interpolation

The Shepard interpolation (Shepard) is an inverse distance weighting method for multivariate interpolation.

The assigned values to an unknown point $\mathbf{m}$ corresponding to a probability vector $\mathbf{p}$ is calculated with a weighted average of the amplitude values available at the known points $\mathbf{m}^j$.

The i-th component of the Shepard interpolator for $i = 1, \dots, p$ is given by:

$$s_i(\mathbf{m}) = \frac{\sum_{l=1}^{n} w_l(\mathbf{m}) A_i^l}{\sum_{l=1}^{n} w_l(\mathbf{m})}$$

with $w_l(\mathbf{m})$ the weight associated to the l-th couple, function of the distance between $\mathbf{m}$ and $\mathbf{m}^j$:

$$w_l(\mathbf{m}) = \frac{1}{\sum_{k=1}^{d-1} \left(\mathbf{m}_k - \mathbf{m}_k^l\right)^2 + \epsilon}$$

where ε is a small number to prevent division by zero, if the distance between $\mathbf{m}$ and $\mathbf{m}^l$ is zero. This interpolation has several features:

i. can be computed in any dimension,
ii. is smooth (differentiable),
iii. is exact, i.e. $s_i\left(\mathbf{m}^j\right) = A_i^j, j = 1, \dots, n, i = 1, \dots, k$,
iv. gives values that do not exceed extreme amplitude values.

However some undesired bumps may occur in some situations where amplitude oscillation is important (Figure 3).

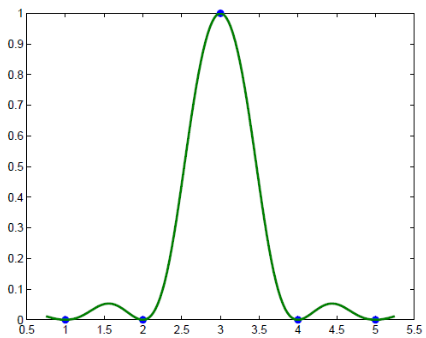

Figure 3: Shepard interpolation with strong variations in input data (in blue). Notice undesired bumps in signal at abscissae 1.5 and 4.5.

To prevent possible artifacts, a band-pass filter may be applied a posteriori to fit seismic frequency content. Here we use a FIR filter implemented recursively in time-domain with constant slope fixed here to 20 dB/decade and user-defined band frequency (we took $f_{low} = 5$Hz and $f_{high} = 35$Hz deduced from seismic). The application is then performed trace by trace and may be applied several times to increase the slope.

Figure 4 shows initial trace gather and result of one iteration smoothing. The effect of global smoothing is noticeable.

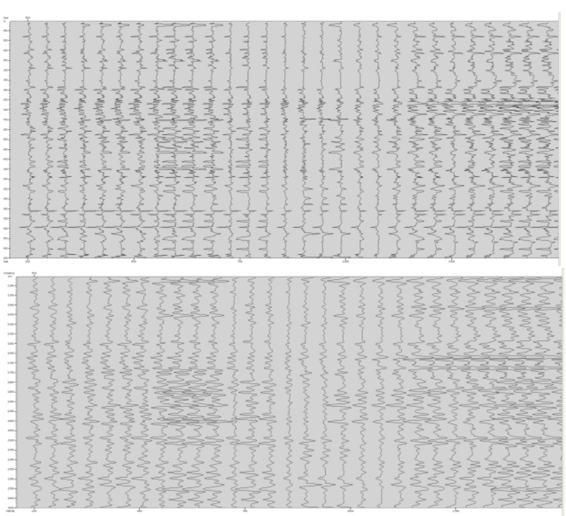

Figure 4: Up: Synthetic CMP gather obtained with Shepard interpolation with 29 offsets. Down: After applying on it the band-pass filter with parameters $f_{low} = 5$Hz and $f_{high} = 35$Hz. We can observe a global smoothing of the signal.

MAIN RESULTS

Input data comes from a supervised classification using DNNA (Hami-Eddine et al., 2011) which gives a link between 17 lithofacies described by probability vectors and corresponding amplitude gather amplitudes. To check the quality of the forward modeling we compare it to the full stack data volume.

Figure 5 shows frequency content of original seismic (red curve), forward modeling (blue curve) and smoothed forward modeling (green curve).

We notice that frequency content of the amplitude forward modeling is wider than the one of the full stack volume. Thus, we have to filter the result to get rid of those added frequencies.

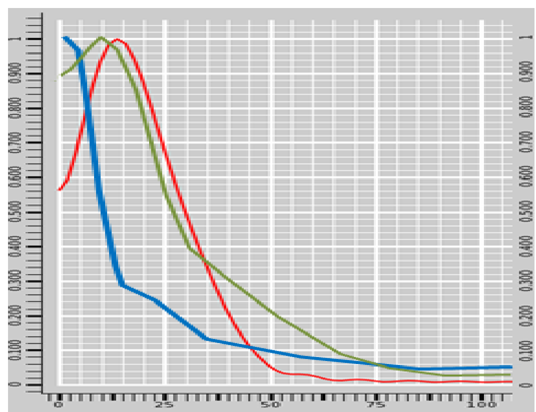

Figure 5: Frequency content of original post stack amplitude (red), stacked forward modeling amplitude (blue) and stacked smoothed forward modeling amplitude (green) for the central rectangular zone displayed in Figure 6.

Results are shown on a seismic section of stacked amplitudes. Figure 6-(a) shows the full stack reference section. Figure 6-(b) and 6-(c) show respectively the application of the forward modeling (respectively smoothed forward modeling).

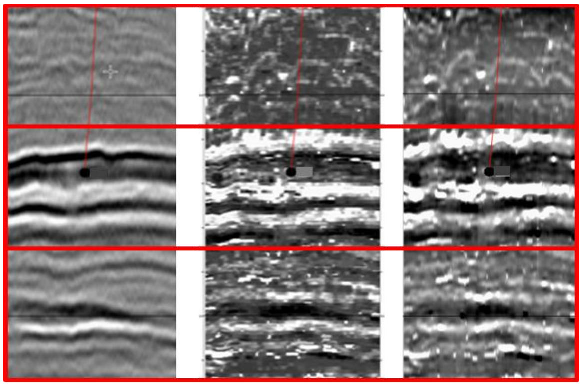

Figure 6: Result of the forward modeling. (a) Full stack reference data. (b) The forward modeling. (c) Smoothed forward modeling.

In the upper part, the original stack shows low contrast, poor continuity and high frequency. The modeled seismic is consistent with the original stack.

In the middle part, the original stack shows a high contrast, an excellent continuity and low frequency. The first event provides intermediate frequencies. Other seismic events show single to double phase. The smoothed forward modeling shows continuous marker with high contrast consistent with the original stack. The first event is extremely low frequency compared to the original stack. Other seismic events are consistent with original stack.

In the bottom part, the original seismic shows intermediate contrast, good continuity, and intermediate frequency content. The smoothed forward modeling shows intermediate contrast which sometimes is inconsistent with original seismic. The continuity is not as good as expected but shows some similarity with the original seismic. The frequency content is globally consistent with original seismic.

CONCLUSION

The joint use of Aitchison transform and Shepard interpolation for forward modeling of pre-stack amplitudes gives interesting results in terms of amplitude reconstruction and frequency content. However spatial continuity needs to be improved. We plan to use this technique for lithofacies inversion. Another application field is reservoir characterization and supervised classification results quality control.

Further improvement directions would consist of spatial continuity improvement, and extension to 3D neighborhood using stratigraphy information.

ACKNOWLEDGMENTS

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