

# Time-varying $Q$ estimation on reflection seismic data in the presence of amplitude variations

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## SUMMARY

$Q$  estimation is an essential step towards achieving a better signal resolution through  $Q$  compensation (inverse  $Q$ -filtering). Although important, seismic  $Q$  is difficult to estimate, hence, a rarely measured quantity. Time-varying  $Q$  functions are especially challenging to estimate and the literature on the subject is limited.  $Q$  estimation in the presence of time-varying amplitude effects such as transmission loss is also seldom discussed in the literature. In this paper, we address both challenges. We build on the work of Wang (2004) and propose three methods for  $Q$  analysis based on different assumptions about the seismic data. In the first approach, we solve the constant  $Q$  estimation problem with a non-linear least-square formulation. Second, we develop a recursive strategy for estimating effective  $Q$  functions (time-varying  $Q$ ). Finally, we devise another method to address the problem of amplitude variations in the seismic data. We use synthetic data to show that our methods achieve accurate results.

## INTRODUCTION

As a seismic wavelet travels the subsurface of the earth, its energy is absorbed and its higher frequencies are attenuated. This absorption effect is quantified by the attenuation factor  $Q$ . An accurate measurement of  $Q$  factor is essential for resolution enhancement i.e., absorption loss  $Q$  compensation, Futterman (1962), Wang (2009).  $Q$  can also be used in reservoir characterization Li et al. (2015).

Reliable estimation of  $Q$  is not trivial. Various methods have been proposed in the literature attempting to achieve a robust  $Q$  analysis. Cheng and Margrave (2012) use a time-domain modification of the spectral ratio method by finding the  $Q$  match filter between wavelets at two different time points and using a direct search method to find  $Q$ . Sun et al. (2014) use regression analysis by leveraging the relation of the spectral ratio slope to estimate  $Q$  in CMP gathers. Zhang and Ulrych (2002) measure the shift of a central or peak frequency from a reference frequency in different wavelets to estimate  $Q$ . Li et al. (2015) use a modified version of the frequency shift method and address the multi-layered (time-varying  $Q$ ) case. Wang (2004) and Wang (2014) proposed two new analysis methods for  $Q$ . They map the time-frequency spectrum to a 1-D spectrum and use data fitting with linear and non-linear least squares to estimate  $Q$ . They extend the constant  $Q$  analysis to the time-varying case. In this paper, we argue that their method does not result in accurate  $Q$  estimates. We propose an improvement over the constant  $Q$  and the multi-layered  $Q$  analysis discussed in Wang (2004).

In the following section, we describe three methods which address different challenges in three different attenuation models. In the first model, the  $Q$  factor is assumed to be constant across

the whole seismic survey or seismic trace. Hence, we extend the work of Wang (2004) and tackle the estimation problem by using a different minimization approach. In the second model,  $Q$  is time-varying i.e., it is a function of time and can be written as  $Q(t)$ . Here we develop a recursive solution to estimate an effective  $Q$  value at extended time windows (layers). Unlike the previous models, the third model is more generic and takes into account the amplitude variation in time. The third method uses a 1-D non-linear data fitting approach over the range of frequencies to compute  $Q$  values at any user defined time locations.

## METHODS

The time-frequency decomposition  $S(\tau, \omega)$  of a seismic trace  $s(t)$  in the presence of anelastic attenuation ( $Q$ ) and time-varying amplitude can be expressed as follows:

$$S(\tau, \omega) = W(\omega) \times A(\tau) \times A_0 \exp\left(\frac{-\omega\tau}{2Q(\tau)}\right), \quad (1)$$

where the  $W(\omega)$  represents the frequency spectrum of the underlying wavelet,  $A(\tau)$  corresponds to the time-varying amplitude effect, and  $A_0 \exp\left(\frac{-\omega\tau}{2Q(\tau)}\right)$  defines the time-varying  $Q$  contribution. It is important to note that the wavelet term is independent of time and the time-varying amplitude effect is independent of frequency.

In practice, the estimation of  $Q$  is performed over a range of frequencies where the amplitude spectrum  $W(\omega)$  is assumed to be constant. In the rest of this paper, we will assume  $\omega \in [\omega_{low}, \omega_{high}]$  and  $W(\omega) = 1$ .

In the following, we present three attenuation models and methods for representing and estimating  $Q$ .

### Constant $Q$ and Constant Amplitude

In this model, we assume the  $Q$  factor and the time-dependent amplitude factor in equation (1) to be constant i.e.,  $A(\tau) = A$  and  $Q(\tau) = Q$  which reduces equation (1) to:

$$S(\tau, \omega) = A' \exp\left(\frac{-\omega\tau}{2Q}\right), \quad (2)$$

where  $A' = A \times A_0$ .

Wang (2004) proposed two methods to estimate a constant  $Q$  by first transforming the time-frequency decomposition to 1-D signal and then estimating  $Q$  by either linear least squares or non-linear optimization. In the linear least squares method, Wang (2004) fits a line to the logarithm of the transformed 1-D signal where the slope represents  $Q^{-1}$ . This technique is unstable and suffers from noise, especially at the end of the spectrum where the amplitude level is below the noise level. In the second method, minimization of the absolute deviation from the compensation function is proposed. This method is more stable, however, tuning the compensation parameter  $\sigma$  for an optimal  $Q$  may not be trivial.

## Time-varying $Q$ estimation

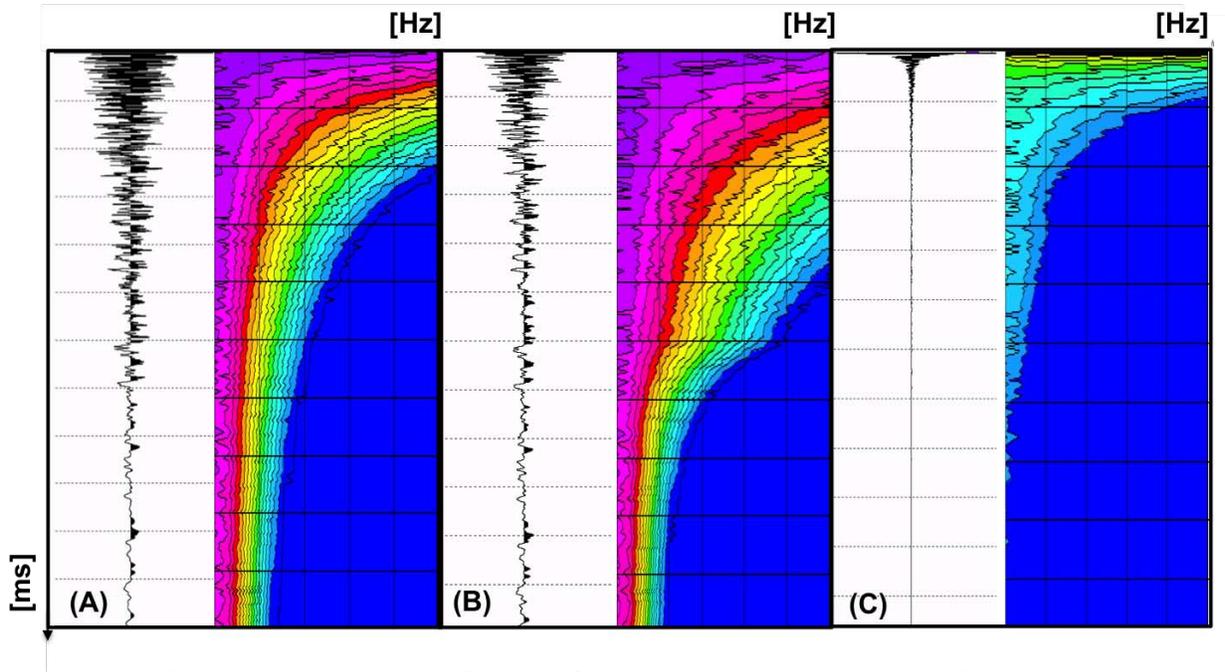


Figure 1: Illustrates effective  $Q$  factor modeling on synthetic traces with 6 seconds length and 2 milli-seconds sampling rate. Panel (A) shows a constant  $Q = 150$  application, and the corresponding time-frequency spectrum. (B) Shows a trace with a  $Q$  function at two time points  $t = 3$  seconds,  $Q = 280$ , and  $t = 6$  seconds,  $Q = 100$ . Panel (C) shows the same trace in (B) with an added spherical divergence effect.

We propose a direct non-linear least square optimization approach over the full 2-D spectrum. Given an estimated time-frequency spectrum  $\hat{S}(\tau, \omega)$  and the theoretical model in equation (2), we define the minimization problem as follows:

$$\arg \min_{Q, A'} \sum_{\omega} \sum_{\tau} (\hat{S}(\tau, \omega) - A' \exp(-\frac{\omega\tau}{2Q}))^2, \quad (3)$$

where we minimize the least square error over the pair  $(Q, A')$ . This approach does not require an additional transformation or an external parameter and is robust since it uses all the data points in the full 2-D spectrum for regression.

The limitation of this method is in the assumption that the amplitude  $A(t)$  is constant. The implication is that all the spherical divergence and transmission losses have been accounted for and the only factor impacting the amplitude variation as a function of time and frequency is  $Q$ . Such requirement is very hard to fulfill in the preprocessing stages since both spherical divergence and reflection/transmission loss compensations are approximate corrections. Because this analysis attributes all amplitude changes to  $Q$ , different spherical divergence and/or gain corrections lead to varying  $Q$  estimates.

### Time-Varying $Q$ and Constant Amplitude

In this method, we will assume that the amplitude effect is the only constant. Hence, equation (1) can be re-written as:

$$S(\tau, \omega) = A' \exp(-\frac{\omega\tau}{2Q(\tau)}), \quad (4)$$

In order to devise a strategy for estimating an effective  $Q$  function  $Q(\tau)$ , we need to define the relationship between effective  $Q$  and interval  $q$  values. Let us assume that the earth is composed of  $N$  layers with different interval  $q$  values,  $q \in \{q_1, q_2, \dots, q_n\}$ . Following Wang (2004), the effective  $Q$  values,  $Q \in \{Q_1, Q_2, \dots, Q_n\}$ , can be expressed as follow:

$$\begin{aligned} Q_1 &= q_1, \\ \frac{1}{Q_n} &= \frac{1}{T} \sum_{i=1}^n \frac{\Delta t_i}{q_i}, \\ &= \frac{1}{T} \left( \frac{1}{Q_{n-1}} + \frac{\Delta t_n}{q_n} \right), \end{aligned} \quad (5)$$

where  $\Delta t_i$  represents the time interval of the  $i^{th}$  layer.

Wang (2004) suggests that simply repeating the analysis for constant  $Q$  estimation in different layers would result in the effective  $Q$  function. We believe that their approach cannot achieve accurate estimates because the 1-D transformation ( $\chi = \omega \tau$ ) is not valid when  $Q$  is a function of time since the transformation will not result in a line (in logarithmic space) with a constant slope  $-\frac{1}{Q}$ :

$$\log(A(\chi)) = -\frac{1}{Q(\tau)} \chi + \log(A_0). \quad (6)$$

Figure 2 shows the estimation of the effective  $Q$  value at 6 seconds from the spectrum in Figure 1 (B) using the method pro-

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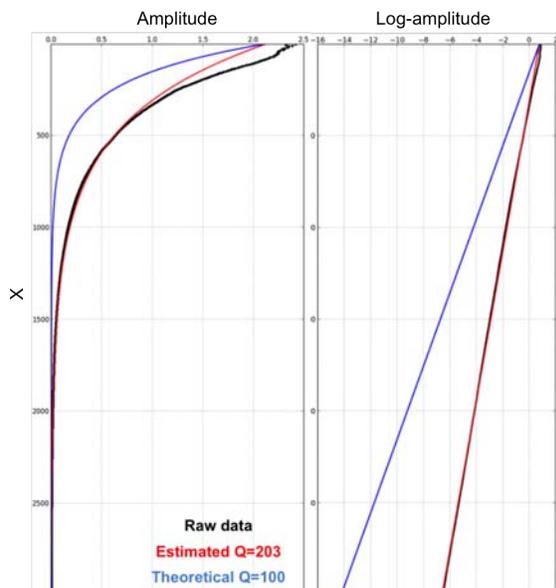


Figure 2: Illustrates the amplitude versus the  $\chi = \omega \tau$  axis based on the estimation method proposed in (Wang (2004)) for time-varying  $Q$ . The black line in the left panel represents the 2-D mapping of the time-frequency spectrum in Figure 1 (B). The red line represents the curve with estimated/fitted  $Q = 203$  and the blue line shows the curve with true  $Q = 100$  value. The right panel plot the amplitudes in logarithmic scale.

posed in Wang (2004). Although, the 1-D transformation results in a curve which resembles a line (in logarithmic space), the estimated value ( $Q = 203$ ) from the data varies largely from the true value ( $Q = 100$ ).

On the other hand, the approach we proposed for constant  $Q$  analysis is still valid if we modify the minimization in equation (3) to be over  $Q(\tau)$  i.e., minimization over a vector of  $Q$  values, rather than the 1D transform suggested by Wang (2004). Such a large problem can be challenging, therefore, we develop a recursive estimation approach. The first value  $Q_1$  is estimated using the constant estimation method. The values afterwards,  $Q_n$ , can be estimated by combining equation (2) and (5) as follows:

$$\begin{aligned} S(\tau, \omega) &= A' \exp\left(\frac{-\omega\tau}{2Q_n}\right), \\ &= A' \exp\left(\frac{-\omega \sum_{i=1}^{n-1} \Delta t_{i-1}}{2Q_{n-1}}\right) \exp\left(\frac{-\omega\tau'}{2q_n}\right), \end{aligned} \quad (7)$$

where  $\tau = \sum_{i=1}^{n-1} \Delta t_{i-1} + \tau'$ , and  $\tau' \in [0, \Delta t_n]$ . The only unknown in equation (7) is  $q_n$ . We substitute equation (7) into equation (3) and formulate the minimization problem as follows:

$$\begin{aligned} &\arg \min_{Q_n, A'} \sum_{\omega} \sum_{\tau} (\hat{S}(\tau, \omega) - A' \exp(\frac{-\omega\tau}{2Q_n}))^2 \\ &= \arg \min_{q_n} \sum_{\omega} \sum_{\tau'=0}^{\Delta t_n} (\hat{S}(\sum_{i=1}^{n-1} \Delta t_i + \tau', \omega) - A' \exp(\frac{-\omega\tau'}{2q_n}))^2 + const, \end{aligned} \quad (8)$$

where:

$$const = \sum_{\omega} \sum_{\tau=0}^{\sum_{i=1}^{n-1} \Delta t_i} (\hat{S}(\tau, \omega) - A' \exp(\frac{-\omega\tau}{2Q_{n-1}}))^2, \quad (9)$$

since  $Q_{n-1}$  has already been estimated in the previous step. Similar to the minimization in equation (3), the error function in equation (8) can be minimized by any non-linear least square solver. The result of using this method on the time-frequency spectrum in Figure 1 (B) is shown in the left panel of Figure 3 (estimated  $Q$  function is in red).

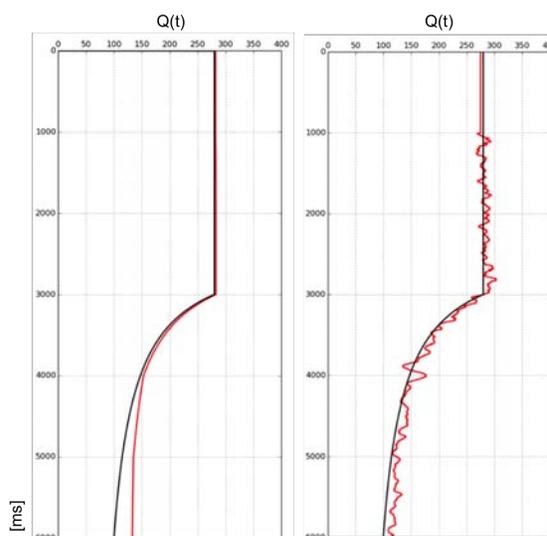


Figure 3: Shows the time-varying  $Q$  using the constant amplitude method (left) and variable amplitude method (right) with the time-frequency spectrum in Figure 1 (B). The black curve represents the true effective  $Q$  function. The red curves display the estimated  $Q$  values. The left curve consists interpolated  $Q$  values from two estimated values at 3 and 6 seconds and interpolated.

### Time-Varying $Q$ and Time-Varying Amplitude

The situation of time-varying  $Q$  and amplitude is by far one of the most encountered cases in seismic data. Amplitude variations can be due to various effects such as spherical divergence and transmission loss. Even if these factors are addressed in prior processing steps, they may not be properly corrected, and any attempt by the previous methods to estimate  $Q$  will result in very low values since the amplitude attenuation will be seen as absorption effect. The reason is the previous approaches use models which assume a constant amplitude over time. We take a different approach for this case. This method

## Time-varying $Q$ estimation

uses the generic model in equation (1) where the amplitude term  $A(\tau)$  is a function of time. Therefore, we need to estimate both unknowns i.e., the effective function  $Q(\tau)$  and the amplitude function  $A(\tau)$  at every time point  $\tau$ :

$$\arg \min_{Q(\tau), A(\tau)} \sum_{\omega} (\hat{S}(\tau, \omega) - A(\tau) \exp(\frac{-\omega\tau}{2Q(\tau)}))^2. \quad (10)$$

In practice, this method suffers from noisy measurements since we are only fitting a cross section of the time-frequency spectrum. To alleviate this issue, a small window around  $\tau$  is used with the constant  $Q$  method to estimate the effective  $Q$  value at  $\tau$ .

It is important to note that this approach is generic and can be used in any situation (see Figure 3). Figure 4 shows the results of this approach compared with constant amplitude model method. The constant method attributes the amplitude attenuation to absorption, thus, results in lower values of  $Q$ .

In the next section, we discuss the advantages and disadvantages of each approach and note what can be done in order to achieve better  $Q$  estimates.

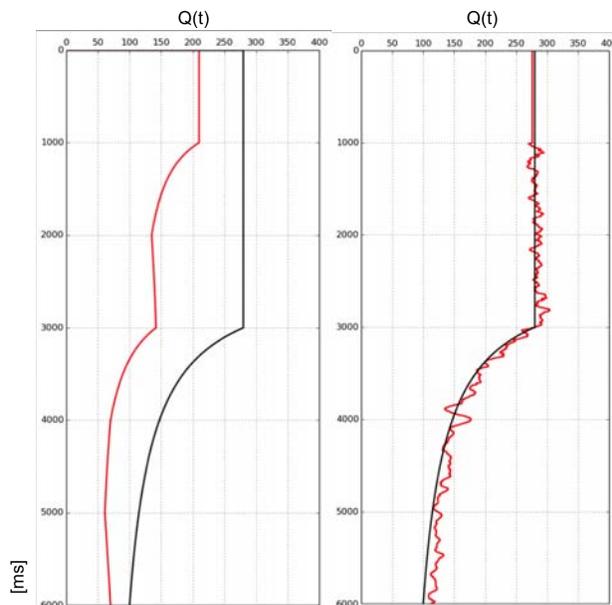


Figure 4: Shows the time-varying  $Q$  using the constant amplitude method (left) and variable amplitude method (right) with the time-frequency spectrum in Figure 1 (C) i.e., with variable amplitude. The black curve represents the true effective  $Q$  function. The red curves display the estimated  $Q$  values.

## CONCLUSION

We have introduced three methods to estimate the anelastic attenuation factor  $Q$ . Each method is based on a different attenuation model. The first method assumes constant  $Q$  and constant amplitude and estimates  $Q$  by fitting the full 2-D spectrum. This method is robust since it uses many data points

for curve fitting. However, this method is rarely useful since it uses unrealistic assumptions. The second method uses a model which assumes a multi-layered  $Q$  function. We have shown that this method estimates accurate  $Q$  functions. The third method takes into consideration the amplitude variation which is one of the main contributions of this work. It does not require knowledge of layer transitions and can estimate a  $Q$  value at every time point.

## EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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