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Restoration of Complex Three-Dimensional Structural Models Based on the Mathematical GeoChron Framework

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Summary

Three-dimensional model restoration typically requires rock mechanical parameters to be input for every layer in the model, as well as boundary conditions. These may be difficult to evaluate and cumbersome to define in the restoration software.

We present a method to restore three-dimensional structural models with no need for mechanical parameters. Based on the GeoChron mathematical framework, our method uses present day geometry and the uvt deposition coordinates to compute a restoration transformation for each horizon in the initial model, with innovative consistency constraints and validation metrics. Faults are automatically taken into account in the computation with no required post-processing to eliminate gaps and overlaps. No user intervention is required other than providing the input GeoChron model.

Once the restoration transformations are computed, the structural model can be restored at any intermediate state quickly enough for the user to be able to browse its geological history. We present results on a sandbox model which show the potential of the method for interpretation validation.
Introduction

Ever since Chamberlin (1910) described how, by following the Pennsylvania railroad which cuts across folded Devonian sandstone, precise dip measurements at regular locations could be used to piece together the local Appalachian folding tectonic story, structural restoration has proved a fertile topic. As well as understanding local geology and tectonic processes, restoration can help validate interpretations. Dahlstrom (1969) laid out the ground rules for cross section restoration in the marginal part of an orogenic belt with the belief that a cross section built with those rules in mind, so that it is possible to restore it, is likely to be more accurate than a cross section that fails the restoration test.

Numerical modelling has built upon these first pen-and-paper efforts and cross sections (Maerten and Maerten, 2006), three-dimensional surfaces (Maerten and Maerten, 2015) and full three-dimensional structural models (Muron, 2005; Chauvin et al., 2018) can now be restored, with direct benefits on model consistency. Mechanical restoration requires grids that conform to geological structures, i.e. faults and horizons, on which finite-element codes may compute a restoration transformation, using physical attributes of the different layers in the model, user-defined boundary conditions and mechanical considerations (Moretti et al., 2006). Using an implicit representation for horizons (Durand-Riard et al., 2010; Chauvin et al., 2018) relieves constraints on the mesh required by finite-element methods and enables mechanical restoration of complex structures. These techniques provide structural model validation tools and workflows (Moretti, 2008) but they may still be too complex to be routinely used in the oil and gas industry.

Making full use of the implicit approach, the \((u, v, t)\) space defined by the GeoChron framework (Mallet, 2004; Moyen, 2005) provides a glimpse at geological structures as they were deposited but does not map the geological space to a true depositional space as the vertical dimension is a geological time, not deposition depth. Mallet (2014) suggested that a new generation of restoration techniques could be derived from this mathematical framework and Lovely et al. (2018) presented the very first. Based on algorithms available in the SKUA® software distributed by Paradigm - Emerson E&P Software, Lovely et al. (2018) adapted the transformation from geological to depositional space to provide a restoration that would preserve layer thickness.

In this paper, we present a full adaptation of the mathematical GeoChron framework which enables three-dimensional restoration of complex structural models with no need for user input. Our new equations address problems pointed out by Lovely et al. (2018) and present mathematically sound solutions illustrated by sequential restoration of an extensional sandbox model.

Theory

The GeoChron-Based Restoration (GBR) method presented in this paper consists in, for each horizon \(H_t\) deposited at geological time \(\tau\), computing a three-dimensional transformation which restores horizon \(H_t\) to its geometry at the time \(\tau\) it was deposited, i.e. the geometry of the sea floor, and passively deforms the whole part of the subsurface that is older than horizon \(H_t\) in a consistent way. Ensuring consistency implies honouring a tectonic style particular to the area being studied, not generating any gaps or overlaps in the restored model and minimising horizon area and layer volume variations.

Figure 1 (A) shows the restored space \(\overline{G}_\tau\) at time \(\tau\), containing a direct frame of orthogonal unit vectors \(\{\mathbf{F}_u, \mathbf{F}_v, \mathbf{F}_t\}\) and a family of horizontal planes \(\{\overline{S}_z(d) : d \geq 0\}\) parallel to \(\{\mathbf{F}_u, \mathbf{F}_v, \mathbf{F}_t\}\). Under tectonic forces, \(\overline{G}_\tau\) deforms into geological space \(G_\tau\) (B) which contains a direct frame of orthogonal unit vectors \(\{\mathbf{r}_u, \mathbf{r}_v, \mathbf{r}_t\}\) and a family of curved surfaces \(\{S_z(d) : d \geq 0\}\) parallel to horizon \(H_z \equiv S_z(0)\). Including horizons older than \(H_t\) (C, D) shows that \(\overline{G}_\tau\) differs from the GeoChron \((u, v, t)\) space in that thickness of layers older than \(H_t\) is preserved.

Accordingly, the \((u_t, v_t, t_t)\) functions required to build \(\overline{G}_\tau\) from \(G_\tau\) differ from the \((u, v, t)\) functions at the heart of the GeoChron framework. First of all, \(t_t\) must be constrained so that restored horizon \(H_t\) in \(G_\tau\) coincides with the sea floor \(S_z(0)\) which, at geological time \(\tau\), is considered to be a continuous,
Figure 1 Principle of GeoChron-Based Restoration (GBR) - Minimal deformation tectonic style. A 
$(u\tau v\tau t\tau)$-transform links the present-day geological space $G_\tau$ (B, C) to the restored space at time $\tau$ denoted $G'_\tau$ (A, D), in which reference horizon $H_\tau$ is restored to its geometry at deposition time $\tau$ and the thickness of the older layers is preserved (D).

unfaulted surface with given topography $z_\tau$ as a function of the GeoChron model $u, v$ coordinates:

$$t_\tau(r_{H}) = z_\tau^o(u(r_{H}), v(r_{H})) \quad \forall r_{H} \in H_\tau.$$  \hspace{1cm} (1)

Next, in order to minimise deformation and volume variation, surfaces $\{S_\tau(d) : d \geq 0\}$ must be level sets of function $t_\tau$ and, for any infinitely small increment $\varepsilon$, the thickness of the thin layer between $S_\tau(d)$ and $S_\tau(d + \varepsilon)$ must be, as consistently as possible, equal to $\varepsilon$. In other words, $S_\tau(d)$ and $S_\tau(d + \varepsilon)$ must be as parallel as possible. This leads to the following constraint on $t_\tau$:

$$||\text{grad} t_\tau(r)|| = 1 \quad \forall r \in G_\tau.$$  \hspace{1cm} (2)

This non-linear equation is not straightforward to honour. Lovely et al. (2018) use a first approximation based on the $t$ function of a GeoChron model built from the single horizon $H_\tau$ to restore and apply a post-processing to scale this first approximation "to depth in a manner that preserves volume or vertical thickness in the restored space". This is a local, computationally-intensive process which becomes even more complex close to faults. Instead of a local post-processing, we compute a global correction function $e_\tau$ on $G_\tau$ which, once added to the initial approximation of $t_\tau$, yields a corrected function $t_\tau = t_\tau + e_\tau$ which satisfactorily honours equation 2.

Figure 2 shows the distribution of $||\text{grad} t_\tau||$ over the $G_\tau$ space for a simple model (C) restored to the middle one of three horizons (D) with a marked lateral variation in layer thickness. Histogram (A) is the distribution of $||\text{grad} t_\tau||$ prior correction, as one would obtain from a regular $(u,v,t)$ computation performed on data from the middle horizon alone. This initial version of $t_\tau$ is far from honouring equation 2 in that the mode of $||\text{grad} t_\tau||$ deviates from 1 and is widely dispersed. After adding global correction function $e_\tau$, $t_\tau$ honours equation 2 much more closely (B).

Similarly, restoration functions $(u_\tau,v_\tau)$ are related to the $(u,v)$ functions of the GeoChron framework but requirements differ. In order to ensure consistency, functions $(u,v)$ and $(u_\tau,v_\tau)$ must be identical on horizon $H_\tau$, which leads to the following constraints for each point $r_\tau^o$ on horizon $H_\tau$:

1) $u_\tau(r_\tau^o) = u(r_\tau^o)$  \hspace{1cm} 3) $\text{grad} u_\tau(r_\tau^o) \approx \text{grad} u(r_\tau^o)$
2) $v_\tau(r_\tau^o) = v(r_\tau^o)$  \hspace{1cm} 4) $\text{grad} v_\tau(r_\tau^o) \approx \text{grad} v(r_\tau^o)$.  \hspace{1cm} (3)

According to the tectonic style chosen to model paleo-geographic coordinates $(u,v)$, functions $(u_\tau,v_\tau)$ must be computed such that their associated restoration deformations are minimized. This is achieved
by constraining them to honour the following differential equations for each point $r \in G_\tau$:

\begin{align*}
\text{Minimal deformation:} & \quad \{ \nabla u_\tau \cdot \nabla v_\tau \}_r \simeq 0 \\
\text{Flexural slip:} & \quad \{ \nabla S u_\tau \cdot \nabla S v_\tau \}_r \simeq 0 \\
\end{align*}

where $\nabla S f$ denotes the projection of the gradient of function $f$ on the level sets $S_\tau$ of function $t_\tau$.

Finally, in order to ensure there are no gaps or overlaps in the restored models, at each restoration time step $\tau$ the faults are sorted into two groups: Those that intersect horizon to restore $H_\tau$ are $\tau$-active faults, those that do not intersect horizon to restore $H_\tau$ are $\tau$-inactive faults. Movement along $\tau$-active faults is enabled by the following constraints set on each pair of twin points $(r_F^+, r_F^-)$ located on faces $F^+$ and $F^-$ of a $\tau$-active fault $F$, respectively, which were collocated at time $\tau$:

\begin{align}
1) & \quad u_\tau(r_F^+) \simeq u_\tau(r_F^-) \\
2) & \quad v_\tau(r_F^+) \simeq v_\tau(r_F^-) \\
\text{Movement along $\tau$-inactive faults is disabled by the following constraints set on each pair of mate points (} & \quad (r_F^0, r_F^0)_\tau \text{ collocated on sides $F^+$ and $F^-$ of a $}\tau\text{-inactive fault } F, \text{ respectively:} \\
1) & \quad u_\tau(r_F^0) = u_\tau(r_F^0) \\
2) & \quad v_\tau(r_F^0) = v_\tau(r_F^0) \\
3) & \quad t_\tau(r_F^0) = t_\tau(r_F^0) \\
\end{align}

Application to extensional sandbox model

Figure 3 shows our restoration workflow applied to the extensional sandbox data presented by Chauvin et al. (2018). The restoration functions are first computed for the six interpreted horizons in 26 seconds of user time on a standard workstation and the model can then be shown in any of its restored states with each update requiring under one second of user time. Restoration quality and accuracy, as well as interpretation errors pointed out by our results, shall be discussed in further publications.

Conclusions

We present a full, mathematical adaptation of the GeoChron model principles to three-dimensional restoration. This method enables restoration of complex structural models to any time step with no user interaction. Boundary conditions and consistency constraints are set automatically based on the initial GeoChron functions. Once all time steps are computed, updating the model to any of those restored states is fast enough for the user to scroll through the geological history of the model.
Figure 3 Restoration of extensional sandbox model with six horizons and 25 faults modelled in a grid containing 139,229 tetrahedra. (A) Initial state, before restoration. (B) Restored state for top horizon. (C) Cross section at location of grey plane in perspective views, for initial state, before restoration. (D) Cross section at restored state for top horizon. (E) Cross section at restored state for dark green horizon. Data by courtesy of IFPEN (https://geoanalog.ifpen.fr).

References


