

Calculation of Well Productivity Index in Stochastic Porous Media

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Summary

The productivity index (PI) is an important characteristic of a well, which indicates its production potential. Analytical solutions of well inflow equation are frequently used to calculate PI, however these solutions are obtained under the assumption of reservoir homogeneity. In a heterogeneous reservoir with spatially variable permeability, the use of these analytical solutions leads to errors in PI calculation.

Upscaling is commonly used to calculate an effective permeability of heterogeneous medium and this technique is now applied to solve many reservoir simulation problems. In reservoirs with stochastic permeability the effective permeability is a random variable characterized by its mean value and variance. These statistics can be directly calculated from the solution of well inflow equation in reservoir, which is a partial differential equation with random coefficient. In turn, its solution is treated as the pressure averaged over the ensemble of permeability realizations. The averaged pressure can be represented as infinite perturbation series over permeability fluctuation. In [1] we used Feynman diagrammatic approach to sum this series and to obtain effective reservoir permeability. The calculation of the effective permeability of a stochastically heterogeneous porous medium has been the subject of numerous studies.

In this study we focus on the calculation of the variance of an effective permeability, which represents the error introduced by replacing heterogeneous medium with homogeneous one. We use the approach from [1] to calculate the variance of effective permeability. The knowledge of statistical characteristics of effective permeability allows us to calculate PI.

It is shown that in contrast to the mean effective permeability, its variance depends on the correlation length of permeability field. Semi-analytical expressions for mean effective permeability and for its variance are obtained for lateral and vertical stochastic heterogeneity. These expressions allow PI, well rate and corresponding uncertainties to be easily estimated. The influence of anisotropy, permeability variance and correlation length on the uncertainty in PI is investigated and compared to the results of Monte-Carlo numerical simulation.

[1] Novikov, A.V., Posvyanskii, D.V. The use of Feynman diagrammatic approach for well test analysis in stochastic porous media. *Comput Geosci* (2019). <https://doi.org/10.1007/s10596-019-09880-1>